# Study Guide for Final Exam

December 9, 2015

## 1 Overview

Since the midterm, we have covered Chapters 3, 4, and 5. Here is an outline of the topics we've included:

1. Chapter 3: Solving for the electrostatic potential
   (a) The method of images
   (b) Separation of variables
      i. Cartesian coordinates
      ii. Cylindrical coordinates (see notes for Electric fields in matter)
      iii. Spherical coordinates
   (c) Multipole expansion
2. Chapter 4: Solving for the electromagnetic field in the presence of a dielectric
   (a) Polarization of materials: Displacement, susceptibility and permitivity
   (b) Boundary conditions in the presence of dielectrics
   (c) Field equations in linear media
   (d) Examples
3. Magnetostatics
   (a) Lorentz force law
   (b) Biot-Savart law
   (c) Ampère's law
   (d) Field equations for magnetostatics:
   (e) The vector potential

## 2 The test

I will ask a selection of the following types of question. Formulas will be given.

1. Solve a boundary value problem using a series expansion in an appropriate coordinate system, i.e., one of the following:

$$\begin{aligned} V(x,y,z) &= \sum_{\alpha,\beta} (a_m \cos \alpha x + b_m \sin \alpha x)(c_m \cos \beta y + D_m \sin \beta y)(e_m \cosh \gamma z + f_m \sinh \gamma z) \\ V(\rho,\varphi) &= a + b \ln \rho + \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^l} \right) (C_m \cos m\varphi + D_m \sin m\varphi) \\ V(r,\theta) &= \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta) \end{aligned}$$

The problem may involve a dielectric.

2. Find the electric field of a charge distribution $\rho(\mathbf{x})$ by direction integration using

$$\mathbf{E}(\mathbf{x}_0) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x})(\mathbf{x}_0 - \mathbf{x})}{|\mathbf{x}_0 - \mathbf{x}|^3} d^3x$$

and/or the magnetic field using the Biot-Savart law

$$\mathbf{B}(\mathbf{x}_0) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J} \times (\mathbf{x}_0 - \mathbf{x})}{|\mathbf{x}_0 - \mathbf{x}|^3} d^3x$$

for a given current distribution, $\mathbf{J}(\mathbf{x})$, or the simplified form for given currents

$$\mathbf{B}(\mathbf{x}_0) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l} \times (\mathbf{x}_0 - \mathbf{x})}{|\mathbf{x}_0 - \mathbf{x}|^3}$$

3. Find the electric potential,

$$V(\mathbf{x}_0) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x})}{|\mathbf{x}_0 - \mathbf{x}|} d^3x$$

or the magnetic vector potential,

$$\mathbf{A}(\mathbf{x}_0) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x})}{|\mathbf{x}_0 - \mathbf{x}|} d^3x$$

by direct integration (or possibly the current form,

$$\mathbf{A}(\mathbf{x}_0) = \frac{\mu_0 I}{4\pi} \oint \frac{d\mathbf{l}}{|\mathbf{x}_0 - \mathbf{x}|}$$

and find the corresponding field.

4. Use Ampère's law to find the magnetic field of a symmetric current distribution,

$$\oint_{\mathcal{C}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\mathcal{S}}$$

5. Write charge $\rho(\mathbf{x})$ and current densities, $\mathbf{J}(\mathbf{x})$.

6. Apply the method of images in a highly symmetric situation.

7. Compute an electric or magnetic dipole moment for a given distribution of charge or current.

$$\mathbf{p}_{electric} = \int_V \mathbf{x}\rho\left(\mathbf{x}\right) d^3x$$
$$\mathbf{m}_{magnetic} = I\int_S \hat{\mathbf{n}} d^2x$$

8. Compute the capacitance of a pair of conductors (planes, cylinders or spheres) separated by a dielectric.

$$C = \frac{Q}{\Delta V}$$

   This involves computing $\Delta V$ by any available method.

9. Use the Lorentz force law to find the motion of a charged particle in given fields,

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

   by finding the electromagnetic force and integrating $\mathbf{F} = m\mathbf{a}$.