Exercises

December 7, 2015

1 Charge to mass ratio of the electron

Thomson first measured $\frac{e}{m}$ in 1897 using crossed electric and magnetic fields, using a method similar to the following.

A beam of electrons (with charge, $-e$, the value unknown at the time) and with unknown initial speed moves along the x-axis between the plates of a capacitor through a distance L. The capacitor produces an electric field in the negative y-direction, $\mathbf{E} = -E\hat{\mathbf{j}}$. After leaving the capacitor, the electron beam travels an additional distance D to a screen, and its deflection d in the $+y$ direction is measured.

Now, in addition to the electric field, a constant magnetic field is applied in the z-direction, $\mathbf{B} = B\hat{\mathbf{k}}$. The strength B is adjusted until there is no deflection of the beam.

Express the charge to mass ration of the electron, $\frac{e}{m}$, in terms of E, B, L, D and d.

2 Force on a square loop in a linearly increasing field

A square loop of wire of side L and carrying a steady current I lies in the xy plane, centered at the origin. A non-uniform magnetic field in the z-direction is given by

$$\mathbf{B} = b_0 x^2 \hat{\mathbf{k}}$$

Find the total force on the loop.

3 Current density

Write the current density, $\mathbf{J}$, for the following situations:

1. A uniform total current I flows through a cylindrical wire. The wire has radius R and lies along the z-axis.
2. A nonuniform total current I flows through a cylindrical wire. The wire has radius R and lies along the z-axis, and the current density is proportional to ρ^2.
3. A wire with constant charge per unit length λ lies along the y-axis and moves with velocity $\mathbf{v} = v_0\hat{\mathbf{j}}$.

4 The Biot-Savart law and Ampère's law

Use either the Biot-Savart law or Ampère's law to find the magnetic field produced by each of the following currrents or current densities:

1. A infinite slab (in the x and y directions) of thickness $2a$ about $z = 0$ carries a current density $\mathbf{J} = J\hat{\mathbf{i}}$. Find the magnetic field everywhere.

2. A nonuniform total current I flows through a cylindrical wire. The wire has radius R and lies along the z-axis, and the current density is proportional to ρ^2.
3. The xy-plane is covered with a constant charge density σ, moving with velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$.
4. A rectangular circuit loop of length a and width b, carrying current I is centered on the origin in the xy-plane. Compute the magnetic field at the origin.

5 Moving charged wires (Griffiths, problem 5.12)

Two infinite, parallel wires separated by a distance d each carry a charge per unit length λ. The wires move relative to the lab with speed v in the direction of their length. Therefore, in the lab frame of reference, they carry both current density and total charge. What is the numerical value for v required for the net force on the wires to be zero?

6 Vector potential

Find a vector potential corresponding to a constant magnetic field, $\mathbf{B} = B_0\hat{\mathbf{k}}$.

7 Magnetic dipole moment

Find the magnetic dipole moment of a rectangular circuit loop of length a and width b, carrying current I and centered on the origin in the xy-plane.