Chapter 12 problems

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1 Velocity selector (including Jackson 12.4)

Consider orthogonal electric and magnetic fields with $B > E$,

$$\begin{aligned} \mathbf{E} &= E\hat{\mathbf{i}} \\ \mathbf{B} &= B\hat{\mathbf{j}} \end{aligned}$$

Show that a particle moving with 3-velocity

$$\mathbf{u} = \frac{cE}{B}\hat{\mathbf{k}}$$

experiences no net Lorentz force. Why do we require $E < B$?

Now work Jackson 12.4.

2 Motion of a charged particle in orthogonal electric and magnetic fields (like 12.5(a) in Jackson)

Consider the problem of orthogonal $\mathbf{E}$ and $\mathbf{B}$ fields, with $|\mathbf{B}| > |\mathbf{E}|$. Solve by boosting to a frame in which the electric field vanishes, solving the remaining pure magnetic problem, then boosting back to the original frame.

Specifically, in the original frame O, let

$$\begin{aligned} \mathbf{E} &= E\hat{\mathbf{i}} \\ \mathbf{B} &= B\hat{\mathbf{j}} \end{aligned}$$

and the initial 4-velocity be

$$u^\alpha = \left(u_0^0, 0, u_0^y, u_0^z\right)$$

With $B > E$, a boost of the right magnitude in the z-direction will result in a pure magnetic field in the new frame, $\tilde{O}$. Boost the initial 4-velocity to this frame as well, and solve. Then perform the inverse boost to find the solution in O.

3 Parallel fields (problem 12.6(b) in Jackson)

Find the motion of a particle of charge q which starts from the origin with 4-velocity

$$u^\alpha = \left(u_0^0, u_0^x, 0, 0\right)$$

in parallel, uniform electric and magnetic fields in the z-direction,

$$\begin{aligned} \mathbf{E} &= E\hat{\mathbf{k}} \\ \mathbf{B} &= B\hat{\mathbf{k}} \end{aligned}$$

4 Jackson, problem 12.9