Problem Set 3a

October 15, 2012

1   Hooke's Law

Two masses, m_1, m_2, orbit one another in otherwise empty space, acted on only by an attractive force varying linearly with the distance between them,

$$\mathbf{F} = -kr\hat{\mathbf{r}}$$

1. Show briefly how to reduce this problem to an equivalent one particle system in a plane (i.e., using reduced mass, conserved angular momentum).

2. Find the effective potential and discuss the properties of bound and unbound orbits.

3. Solve the orbit equation for $r(\varphi)$ and use the result to show that bound orbits are closed.

2   Problem 3.13

The circular orbit must pass directly through the center of force, $r = 0$. It is not difficult to write the Cartesian equations of a circle, offset so that the origin lies on the circle. Transform this to (r, φ) coordinates, and you have the orbit equation. Differentiating then gives the potential and the force. Note in part (b) that the energy is only defined up to a constant. You will get $E = 0$ if you take the zero of the potential to lie at infinity.

3   Problem 3.19

This force is closely related to the Yukawa potential,

$$V_{Yukawa} = -\frac{k}{r}e^{-\mu r}$$

also known as the screened Coulomb potential. It gives a good approximation to the electric field experienced by an outer electron in a multielectron atom, and also describes a solution for a massive scalar particle. The potential is also the solution for the Coulomb potential produced by a massive photon.

Unfortunately, the force given in the problem differs from this one by one term, since

$$\mathbf{F}_{Yukawa} = -\left(\frac{k}{r^2} + \frac{\mu k}{r}\right)e^{-\mu r}\hat{\mathbf{r}}$$

while you are asked to study only the first term,

$$\mathbf{F}_{Goldstein} = -\frac{k}{r^2}e^{-\frac{r}{a}}\hat{\mathbf{r}}$$

However, you are only asked to study the orbits qualitatively, and you can do this from the effective potential using basic properties of the Ei function. For the second part of the problem, the discussion of Bertrand's theorem and the force law are all you will need.