Problems: 19, 21, 22, 23, 24abc

Notes

On problems 19, 21, 22 and 24, write and vary the action to obtain the equations of motion.

On problem 23, use the Euler-Lagrange equation with a generalized force

Problem 19:

Reduce the equations of motion to quadratures. This will take the form

$$t = \int_{\theta_0}^{\theta} \frac{\sin\theta d\theta}{\sqrt{f(\theta)}}$$

where $f(\theta)$ is a polynomial in trigonometric functions.

Problem 21:

The first integral is a straightforward energy-type integral. For the second integral, use Wolfram Integrator or some other source to get the integral. Do not try to reduce it further; it is too messy.

Problem 22:

I find that it helps to write the kinetic energy first in terms of Cartesian coordinates, then write those in terms of the two angles.

Problem 23:

Variational methods do not always work when dissipative forces are present, so just use the Euler-Lagrange equation with a generalized force for this one, eq. 1.53. Integrate completely to find $z(t)$ and take $t \longrightarrow \infty$ in your expression for $\dot{z}$ to find the terminal velocity.

Problem 24:

Only the first three parts, a, b, c, are required. The presentation is a bit garbled here. Let L be the length of the spring when the mass M hangs straight down at rest. The motion lies in a vertical plane, and there are two degrees of freedom (an angle, and the stretch of the spring).