

Problem W8

December 5, 2012

Projectile motion in two dimensions

1. Solve the problem of a projectile in the xz -plane, in a constant gravitational field with acceleration, $\mathbf{g} = -g\mathbf{k}$. Find the Lagrangian and Hamiltonian for this system. Identify any conserved quantities. At initial time $t = 0$, let the projectile have initial position $\mathbf{x}_0 = (x_0, z_0)$ and initial momentum $\mathbf{p}_0 = p_{0x}\mathbf{i} + p_{0z}\mathbf{k}$. Solving any way you like, write the solution for $(x(t), z(t))$ in terms of these general initial conditions.
2. Invert this solution to give the initial conditions in terms of the values of x, z, p_x, p_z at time t .
3. Find Hamilton's principal function for this system in three ways: (1) Put the solution into the action and carry out the integral; (2) Integrate the equations relating the generating function to the coordinates and momenta; (3) Solve the Hamilton-Jacobi equation.