Problem Set 1b

September 16, 2014

1. Atwood's machine. A rope passes over a uniform pulley of mass, M, and radius, R. Masses m_1 and m_2, with $m_1 < m_2$, are attached to the ends of the rope. The system rotates toward m_2 with m_1 rising and m_2 falling with the same acceleration a. Take the direction of motion as positive for all forces and velocities. Find the acceleration a.

2. Atwood's machine with massive rope. The same as problem 1, but now let the rope have non-negligible mass. Let the total length of the rope be $L = l + \pi R$ and have mass m. Assume $m + m_1 < m_2$ so the motion is in the same direction as in problem 1. Let the initial position be such that m_2 even with the center of the pulley so that m_1 is a distance l below the center of the pulley. Find the acceleration $a\left(t\right)$. Notice that then tension in the rope is not constant. Show that after simplification, the equations of motion for the system can be reduced to a single equation of the form

$$\alpha + \beta x = \frac{a}{g}$$

where x is the length of rope on the m_2 side of the pulley, and

$$\begin{aligned} M_{eff} &= \frac{1}{2}M + m_2 + m_1 + m \\ \alpha &= \frac{1}{M_{eff}}\left(m_2 - m_1 - \frac{ml}{L}\right) \\ \beta &= \frac{2m}{LM_{eff}} \end{aligned}$$

Integrate this equation to find $x\left(t\right)$.

3. Raindrop in fog. An initially infinitesimally small droplet of water (density ρ_W) falls from rest in a uniform fog (density ρ_F) under the influence of gravity, mg. As the drop falls, it sweeps up water from the fog and grows in size, and hence mass, so that the drop sweeps out a conical section of fog. By solving completely for $x\left(t\right)$, show that the acceleration of the drop is constant and equal to $\frac{1}{7}g$, where g is the usual acceleration of gravity.