Problem Set I.b

September 6, 2018

Problem Set 1: Due Wednesday, September 12

Problems are from Goldstein, second edition

Work Goldstein problems 16, 17, 18, 19, 20 but find the equation of motion *by writing and varying the action instead of going straight to the Euler-Lagrange equation.*

From Goldstein, with modified instructions:

1. Vary the action

$$S = \int \left(\frac{1}{2} m \left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2 \right) - \frac{1}{2} K \left(ax^2 + 2bxy + cy^2 \right) \right) dt$$

   where $b^2 - ac \neq 0$. Vary S with respect to x and y independently to find the two equations of motion. Look at the special cases$a = 0 = c$ and $b = 0, c = -a$. What physical system does this describe? See if you can change variables to find a simpler expression for the action.

2. Study the spherical pendulum: a mass point m is suspended on a rigid, massless rod of length L from a central point, leaving it free to move anywhere on a sphere of radius L. Gravity acts on m. Write and vary the action. Explore solutions for any simple initial conditions.

3. Let

$$S = \int \left(\frac{1}{12} m^2 \dot{x}^4 + m\dot{x}^2 V(x) - V^2(x) \right) dt$$

   where $V(x)$ is differentiable. Vary S to find the equation of motion and solve.

4. Two point masses, m_1 and m_2, are connected by a string of negligible mass and length L. Mass m_1 rests on a horizontal tabletop and is free to move in that plane, but the string passes through a hole in the table and mass m_2 hangs straight down. Find a pair of generalized coordinates for the system, write and vary the action. Find a first integral for the motion (the complete solution is too messy). What quantity is conserved?

5. Consider a double pendulum: a mass m_1 is suspended from the ceiling by a massless cord of length L_1, while a second mass m_2 hangs by a second cord of length L_2 from the first mass. Let the motion be in a single plane containing both masses. Write the action and find the equations of motion. Find any solutions you can.

6. Consider the same double pendulum as in Problem 5, but allow the motion to move in both swinging motions and rotational motion. Find and vary the action. Don't worry about solving unless you want to try some cases.