Problems

November 20, 2018

1. For each of the following actions, find the conjugate momentum for each variable, write the Hamiltonian, and write Hamilton's equations. You do not need to solve.

   (a) In one dimension, $S = \int \left(\frac{1}{2}m\dot{x}^2 - V(x)\right) dt$

   (b) In three dimensions, $S = \int \left(\frac{1}{2}m\dot{\mathbf{x}}^2 - mgz\right) dt$

   (c) $S = \int \left(\frac{1}{2}m\dot{x}^2\dot{y}^2 - kxy\right) dt$

   (d) $S = \int \left(\frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2\right) + \frac{k}{r}\right) dt$

2. Compute the Poisson bracket of f and g where $f(x,p) = x^2p$ and $g = p^2 + x^2$.

3. Let x, p satisfy fundamental Poisson brackets,

$$\begin{aligned}\{x,x\} &= 0\\ \{x,p\} &= 1\\ \{p,p\} &= 0\end{aligned}$$

   Let $q = x^2p^3$. Find an expression for a momentum π conjugate to q by requiring the Poisson brackets

$$\begin{aligned}\{q,q\} &= 0\\ \{q,\pi\} &= 1\\ \{\pi,\pi\} &= 0\end{aligned}$$

   The answer is not unique. Once you have found a suitable π, show that $\pi + f\left(x^2p^3\right)$ is also a solution, where f is any function of x^2p^3.