Problems

October 4, 2018

I have reworded and/or given hints for the problems from Goldstein.

1. The first order perturbation of circular orbits for a bead constrained to move on a paraboloid is now (or will soon be) in the notes. Using the notation there, find the perturbative second order solution.

2. Carry out the perturbation of a circular orbit subject to an inverse-square-law central force. Show that the result gives a closed, elliptical orbit, that is, that the period of small oscillations is the same as the original orbital period.

3. Problem 3.14 and 3.15, Goldstein: Show that the motion of a particle in a potential

$$V(r) = -\frac{k}{r} + \frac{h}{r^2}$$

   is the same as the motion in a Kepler potential alone when expressed in a coordinate system rotating around the center of force. Equivalently, show that the motion is Keplerian with an anomalous precession of the orbit. For bound states ($E < 0$) with $\frac{h}{r_{min}} \ll k$ show that the rate of precession is

$$\dot{\Omega} = \frac{2\pi m h}{l^2 \tau}$$

   where $\tau$is the orbital period without the perturbation. After accounting for the effect of the other planets, the orbit of Mercury has a residual precession as a rate of 40 seconds of arc per century. Show that this could be accounted for by taking $h = ka\eta$ (where η is a small, dimensionless parameter) with η as small as 7×10^{-8}, where the eccentricity of Mercury's orbit is 0.206 and its period is 0.24 year.

4. Problem 3.23, Goldstein: A magnetic monopole is defined (though one has never been detected) by a magnetic field of the form

$$\mathbf{B} = \frac{b}{r^2}\hat{\mathbf{r}}$$

   where b is constant. Suppose a single point source gives rise to both a central electric potential, $V = -\frac{k}{r}$, and this monopole field. A particle of mass m and electric charge q moves in response to the combined forces. Write Newton's equation of motion for the particle, where the force is given by the Lorentz force law,

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

   Because of the cross product, this is not a central force, and angular momentum is not conserved.

   (a) By considering $\mathbf{r} \times \dot{\mathbf{p}}$ show that the vector

$$\mathbf{D} \equiv \mathbf{L} - \frac{qb}{c}\frac{\mathbf{r}}{r}$$

   is conserved.

   (b) Show that there exists a conserved generalization of the Laplace-Runge-Lenz vector, with $\mathbf{D}$ replacing $\mathbf{L}$.