Problems

October 4, 2018

All of these problems are from Goldstein, Chapter 3. I reword or give hints for some of them.

1. Problem 3.1, Goldstein: A perticle of mass m is constrained to move on a paraboloid of revolution whose axis is vertical. Find the 1-dimensional problem equivalent to its motion. What condition on the initial velocity produces circular motion? Find the period of small oscillations about this circular motion.

2. Problem 3.3, Goldstein: Two particles move about each other in circular orbits with period τ, bound by their mutual gravitational force. They are then stopped in place and allowed to fall straight toward one another. Prove that they collide after a time $\frac{\tau}{4\sqrt{2}}$.

3. Problem 3.6, Goldstein:

   (a) Show that if a particle describes a circular orbit under the influence of an attractive central force which is directed toward a point *on* its circular orbit, then the force varies as the inverse fifth power of the distance. Hints: The circular orbit must pass directly through the center of force, r = 0. It is not difficult to write the Cartesian equations of a circle, offset so that the origin lies on the circle. Transform this to (r, φ) coordinates, and you have the orbit equation. From this you can find the potential and the force.

   (b) Show that for the orbit described the total energy of the particle is zero. (Note that the energy is only defined up to a constant. You will get E = 0 if you take the zero of the potential to lie at infinity.)

   (c) Find the period of the motion.

   (d) Find $\dot{x}, \dot{y}$ and v as a function of the angle around the circle and show that all three are instantaneously infinite as the particle passes through the center of force.

4. Express the elliptic equation
$$r = \frac{p}{1 + \epsilon \sin\varphi}$$
where r is the distance to the ellipse from one focus and φ its angle from the x-axis, in Cartesian coordinates centered at the center of the ellipse.

5. Express the solution for the inverse square law central force problem in terms of E and L.

6. Problem 3.7 (optional), Goldstein: This one is extra for Josh, and anyone else who wants to work with elliptic functions. Show that the central force problem is soluble in terms of eliptic functions when the force is a power law function of the distance, r^n, with any of the following fractional exponents:
$$n = -\frac{3}{2}, -\frac{5}{2}, -\frac{1}{3}, -\frac{5}{3}, -\frac{7}{3}$$