

Problems

December 11, 2018

1. Let $S = -\int \left(mc^2 \sqrt{1 - \frac{\dot{x}^2}{c^2}} \right) dt$. Find the conjugate momentum to x , write the Hamiltonian, write Hamilton's equations, and solve them.
2. Are (q, π) canonical variables, where $\pi = xp^2$ and $q = \frac{1}{p}$? Assume that x, p satisfy fundamental Poisson brackets.

$$\begin{aligned}\{x, x\} &= 0 \\ \{x, p\} &= 1 \\ \{p, p\} &= 0\end{aligned}$$

3. A particle of mass m moves in two dimensions (x, z) under the influence of gravity with $V = mgz$. Carry out the following steps to solve for the motion:
 - (a) Write the Lagrangian.
 - (b) Find the conjugate momenta to x and z .
 - (c) Find the Hamiltonian.
 - (d) Write the Hamilton-Jacobi equation.
 - (e) Solve the Hamilton-Jacobi equation for Hamilton's principal function \mathcal{S} . Hint: Try a fully separable function,

$$\mathcal{S} = \mathcal{S}_x(x) + \mathcal{S}_z(z) - Et.$$

(or, if you're treating it as 3-dim,

$$\mathcal{S} = \mathcal{S}_x(x) + \mathcal{S}_y(y) + \mathcal{S}_z(z) - Et.$$

- (f) Perform a canonical transformation using $f = -q^i \pi_i + \mathcal{S}(x^i, t)$ as the generating function.
- (g) Invert the transformation equations to find $x(t), z(t)$ in terms of the initial constants q, π .