The fundamental interactions

The **Standard Model** describes two fundamental interactions:

1. Electroweak
2. Strong

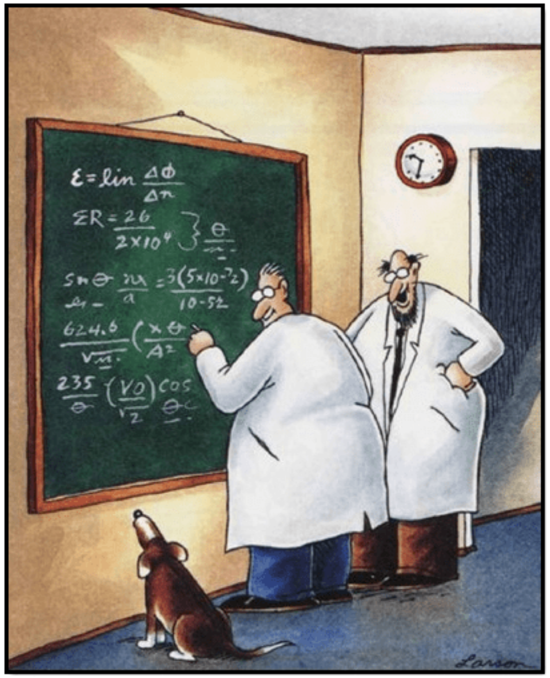
General Relativity describes the third interaction:

3. Gravity

We don't know how to do exact QFT calculations:

- The theories are ***nonlinear***
- The theories are ***quantum***

Therefore we use ***perturbation methods*** in Quantum Field Theory.



"Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics."

Feynman diagrams

Perturbation theory for the electroweak interaction

A principal goal of quantum field theory is to predict, from a given initial particle configuration, what the configuration will be at a later time.

For isolated *single particles* we can solve the problem exactly.

The problem arises when we consider ***interactions***:

- Highly nonlinear
- Involves arbitrarily many fields

In linear problems, we break the total motion of a system into normal modes, and add them. For nonlinear problems, we often can use *perturbation theory* to describe the motion as an infinite series of linear problems.

At each order of perturbation theory, we solve a simpler problem, but the full answer is the sum of all the bits.

Example: the Dirac equation

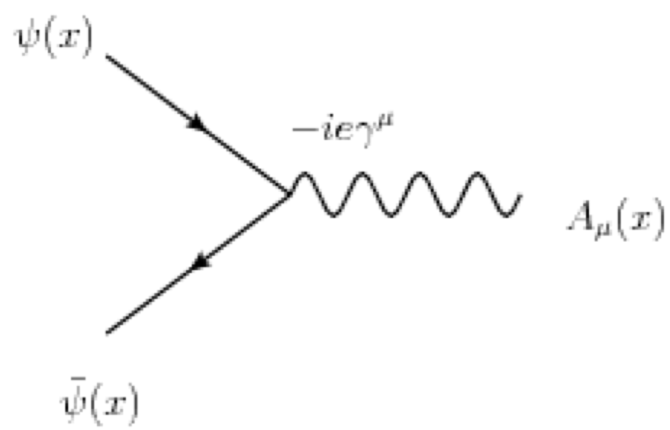
$$i\gamma^{\alpha}\partial_{\alpha}\psi - m\psi = 0$$

We can solve the quantum theory of this equation exactly.

The ***interacting*** quantum Dirac theory looks like this:

$$i\gamma^{\alpha}\partial_{\alpha}\psi - m\psi - ieA_{\alpha}\gamma^{\alpha}\psi = 0$$

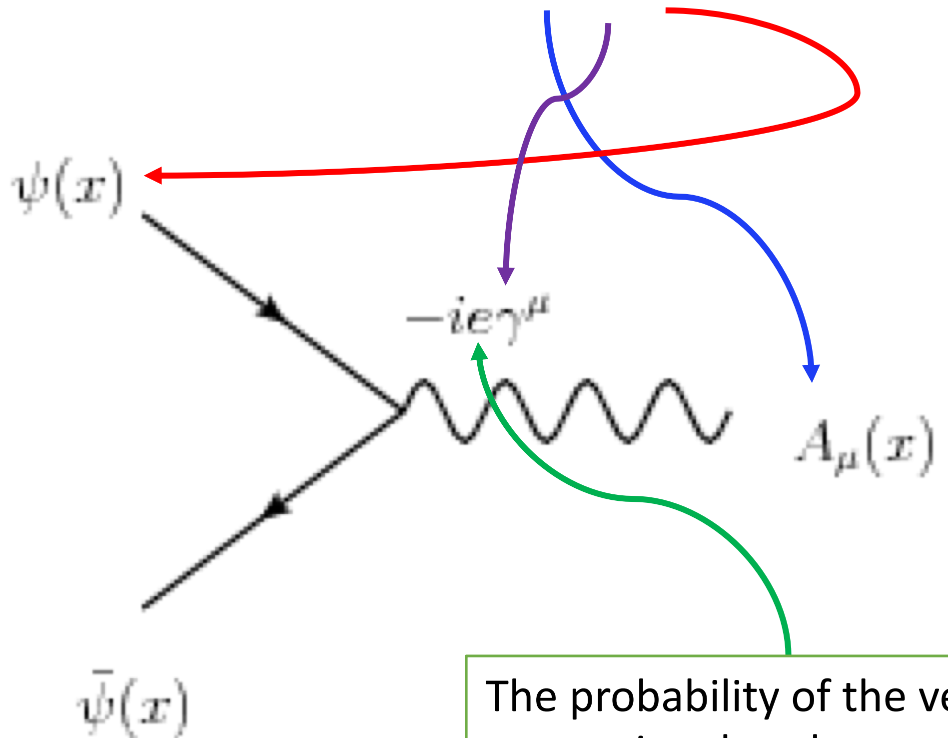
The extra term is drawn as a ***vertex:***



Example: the Dirac equation

Each part of the extra term contributes to the diagram

$$i\gamma^{\alpha}\partial_{\alpha}\psi - m\psi - ieA_{\alpha}\gamma^{\alpha}\psi = 0$$



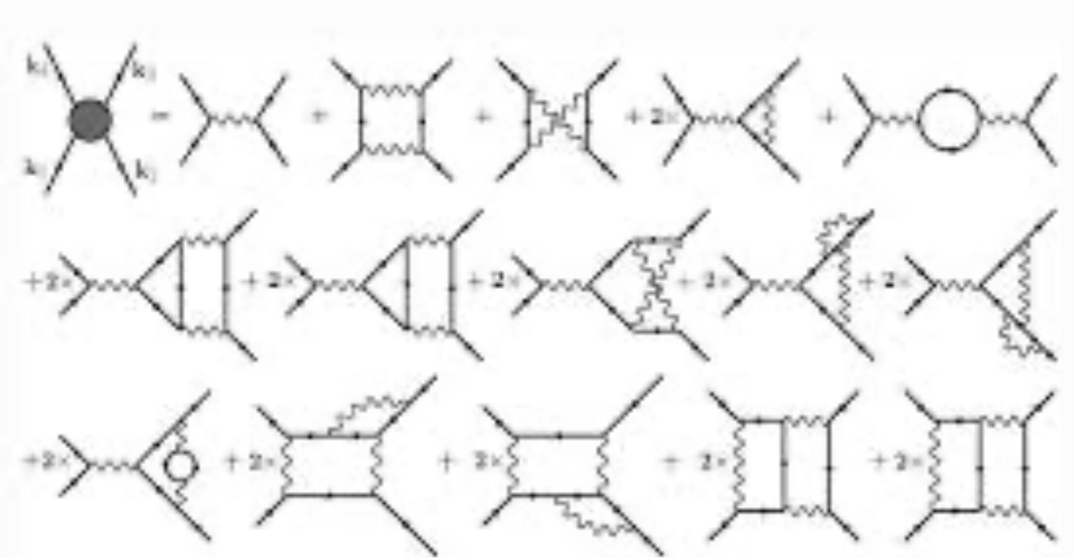
The probability of the vertex occurring is proportional to the ***coupling constant.***

The complexity arises because in the quantum world:

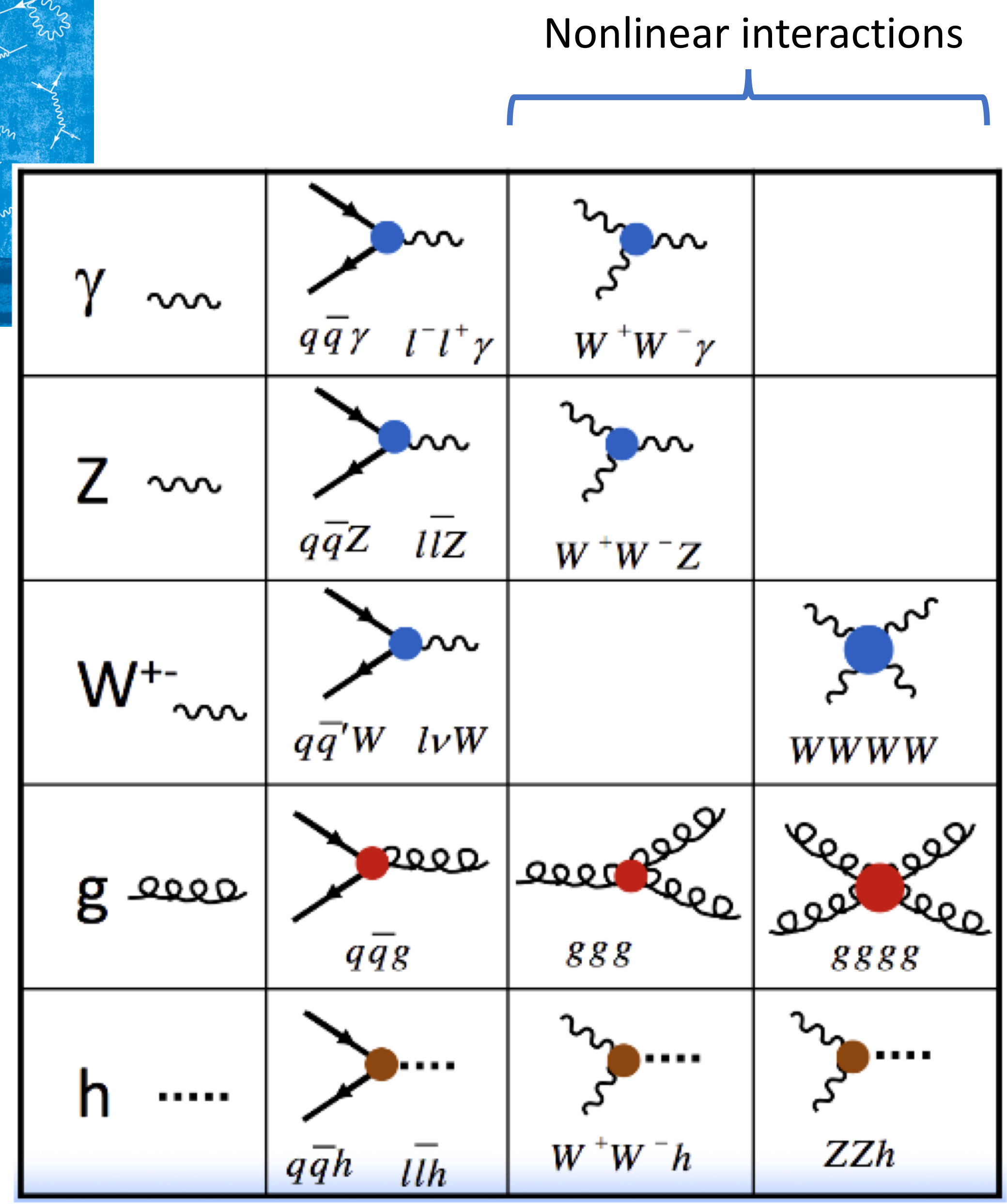
If it can happen, it will

This means that the vertex might occur any number of times before we make a measurement.

We have to consider the (small) probability that things like this happen:



Allowed vertices of the Standard Model



Feynman diagrams

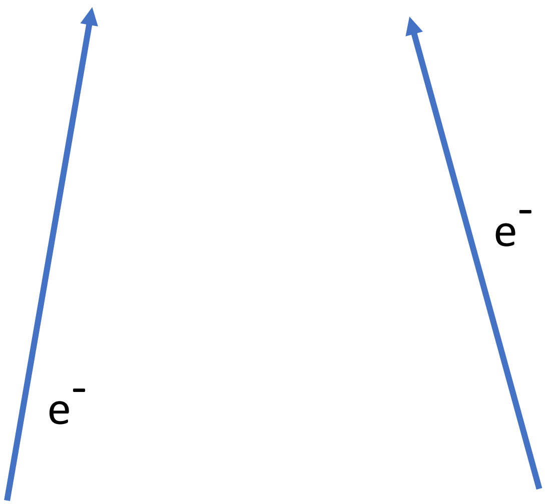
Consider a perturbation approach to two systems:

1. A guitar string via Fourier series
2. A pair of electrons via Feynman diagrams

At lowest (zeroth) order:

Guitar

Electrons

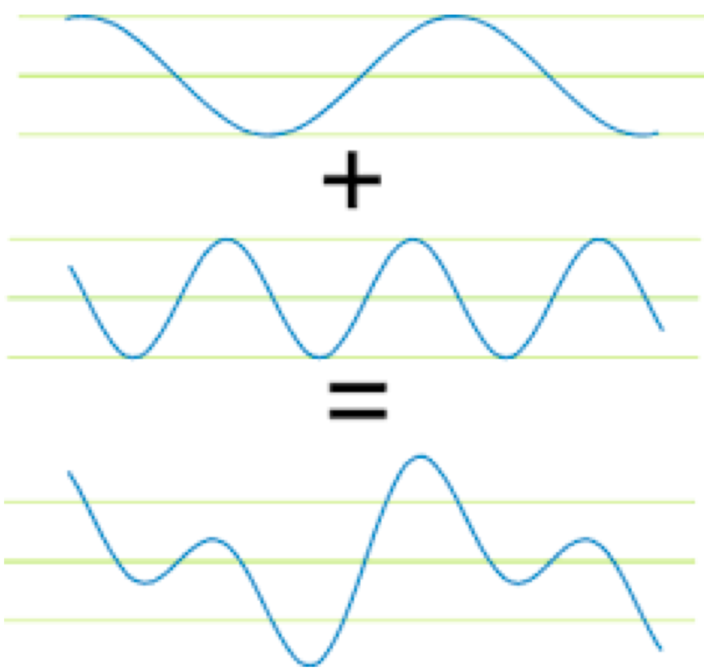


Let's use the language of Feynman diagrams to describe both systems.

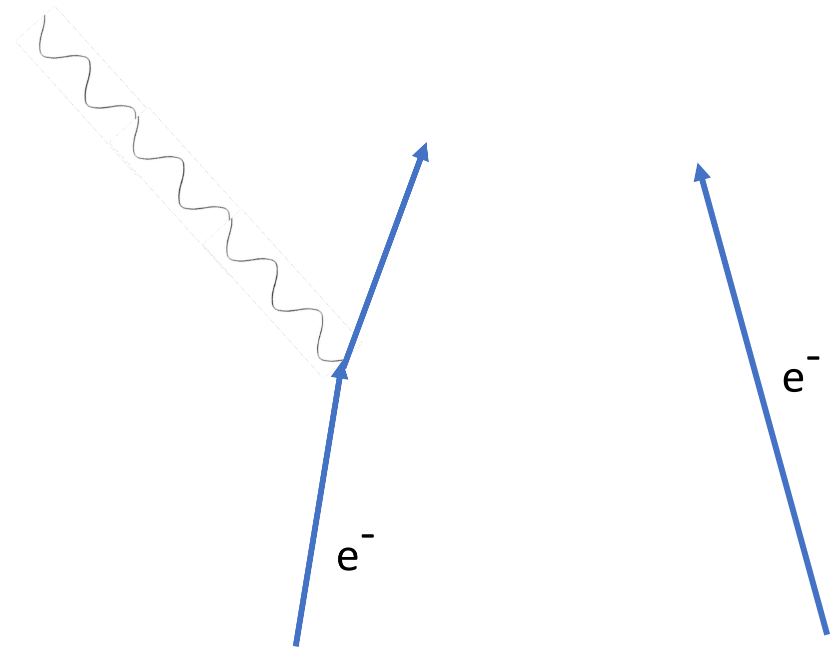
Feynman diagrams

At first order:

Guitar



Electrons



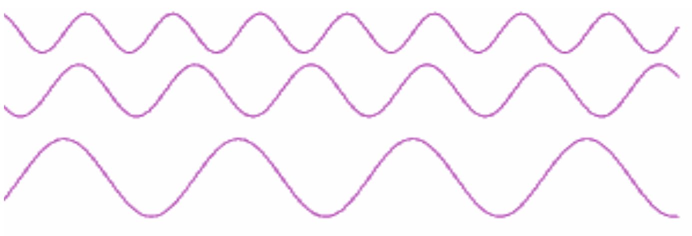
Two sine waves add to give a better approximation to what actually occurs.

Either electron might emit a photon. This changes the final state, so we don't count it.

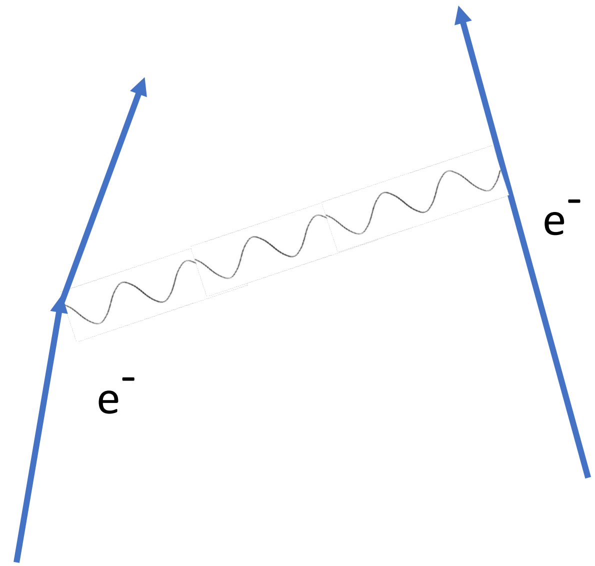
Feynman diagrams

At second order:

Guitar



Electrons



One possibility

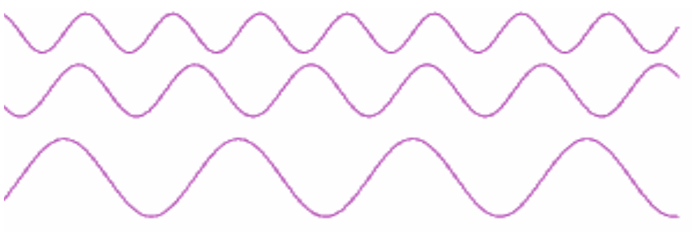
We add in a third sine wave.
Each wave satisfies the boundary conditions.

We add in another vertex, consistent with the boundary conditions (two electrons in, two electrons out).

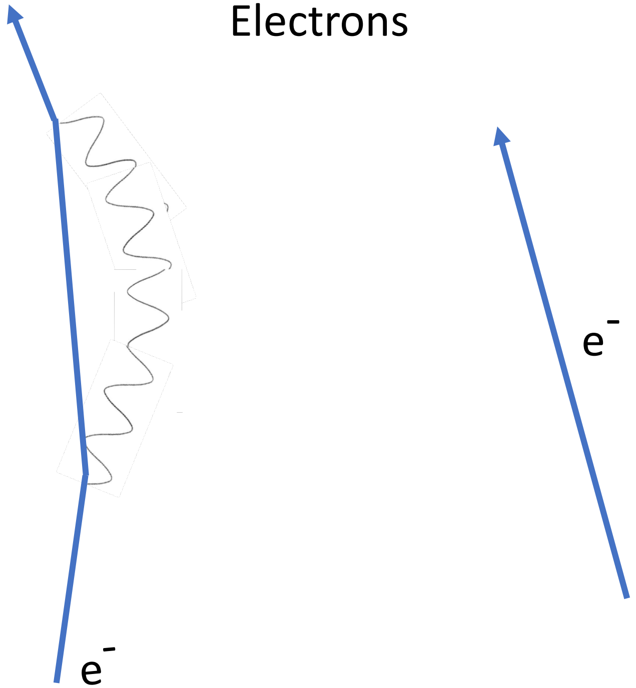
Feynman diagrams

At second order:

Guitar



Electrons



A third possibility has the light on the other electron. We combine all three.

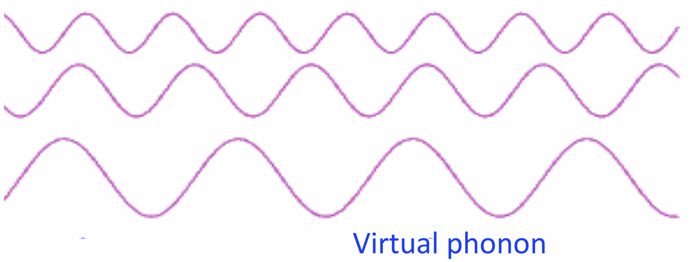
We add in a third sine wave. Each wave satisfies the boundary conditions.

We add another vertex, consistent with the boundary conditions (two electrons in, two electrons out).

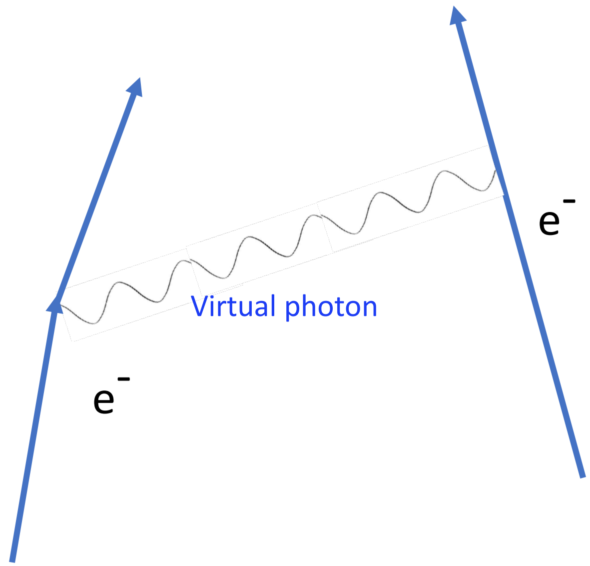
Feynman diagrams

At second order:

Guitar



Electrons



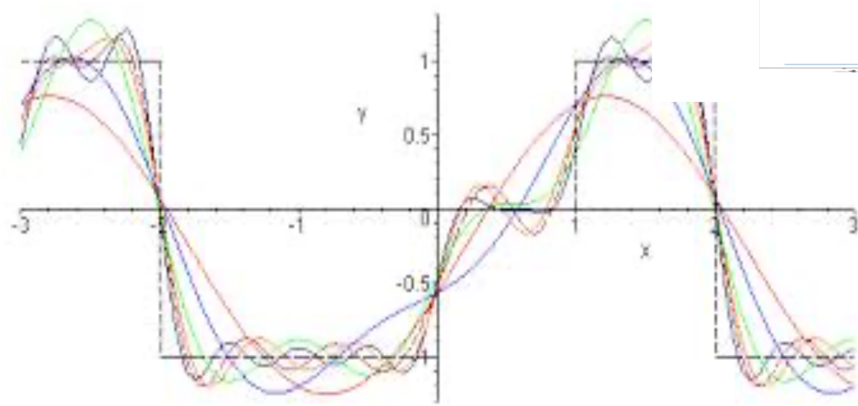
We call the third wave a “virtual particle”

We call the unseen light a “virtual photon”.

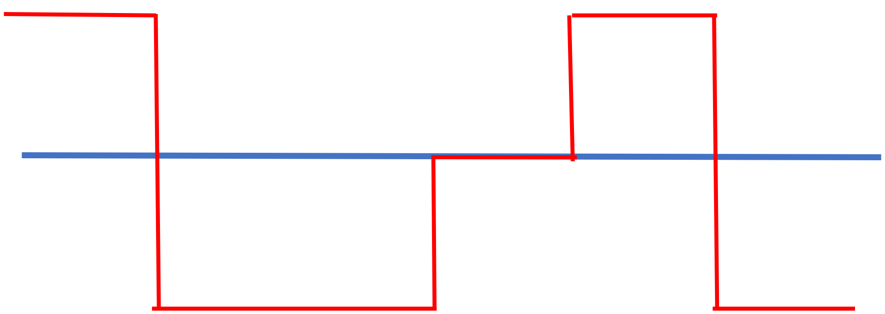
Feynman diagrams

The final description of the process:

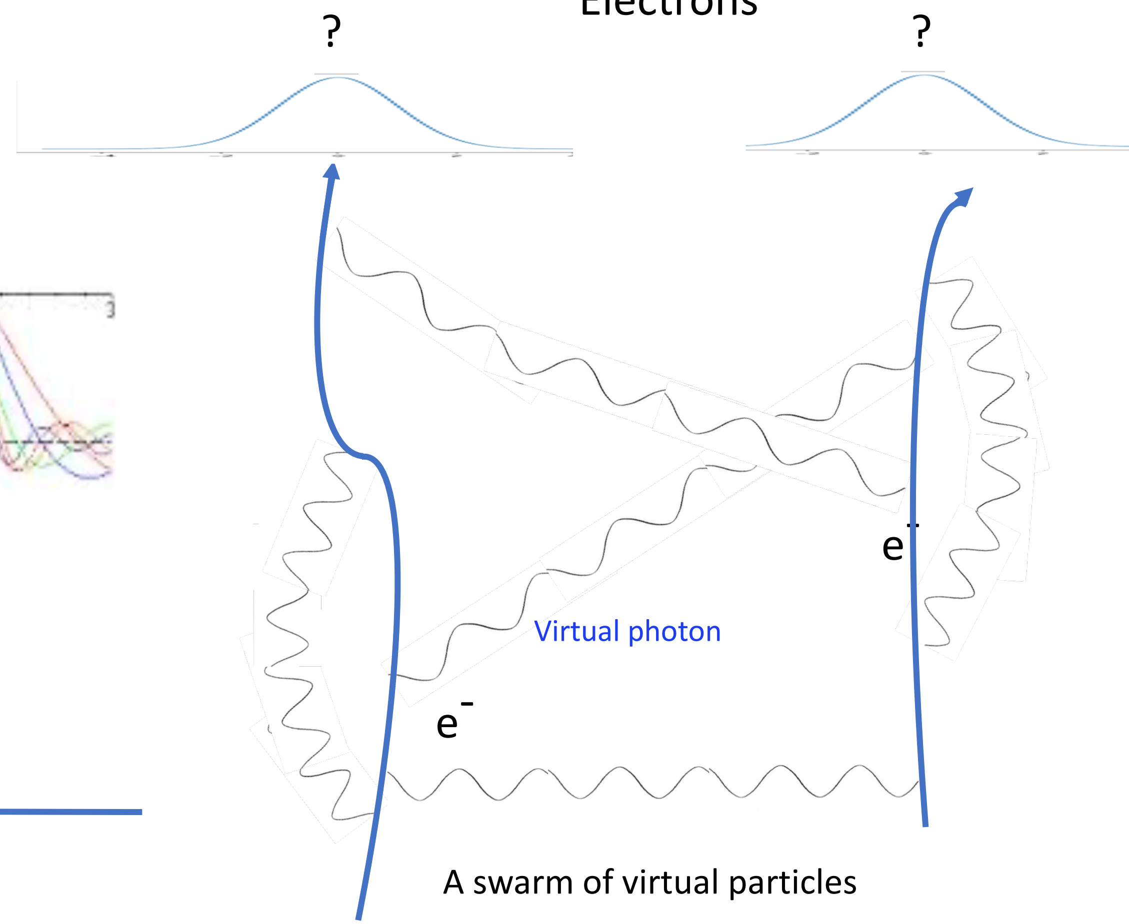
Guitar



A swarm of virtual waves



Electrons



A swarm of virtual particles

The final wave may look a lot different from a sine wave.

The different diagrams place the electron in different positions at the end. We can only predict the probability of the final position.

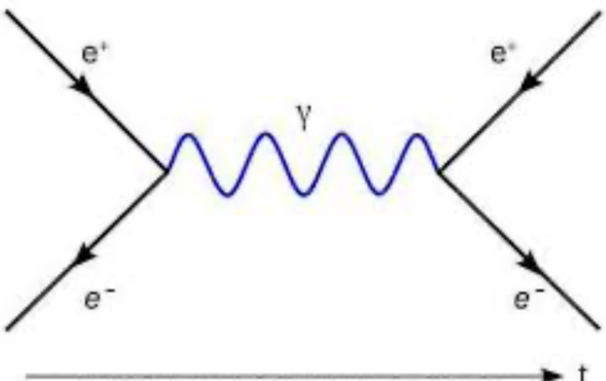
Caveat: Feynman diagrams only work when the interaction is weak enough, then only up to a point.

At each order in perturbation, two things happen:

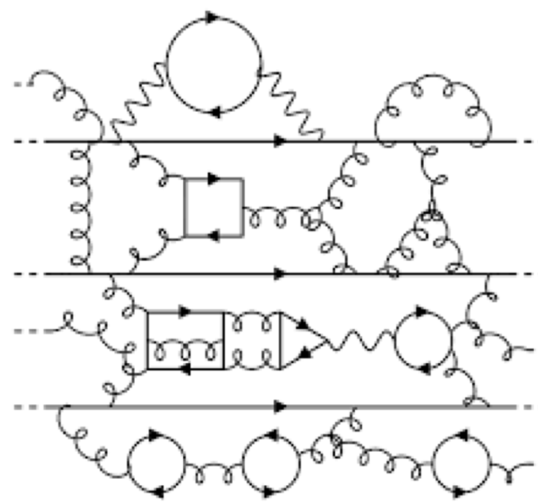
1. The importance of the new terms ***decreases*** by a power of the coupling constant.
   - For electroweak interactions this is the fine structure constant,

     $$\alpha \sim 1/137$$
   - For strong interactions it is about 1.

2. The number of diagrams grows ***very*** rapidly with each added vertex.



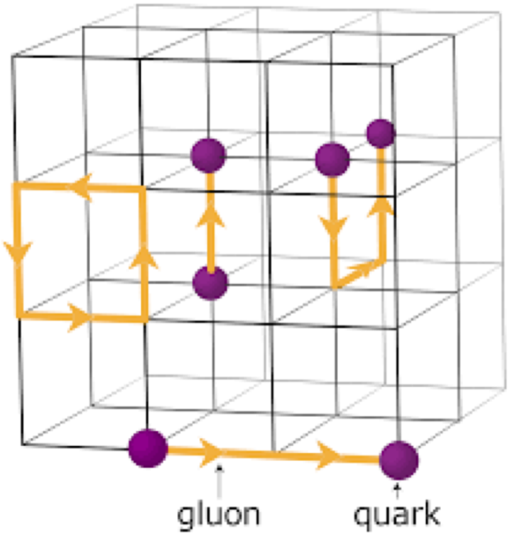
Annihilation of a electron and a positron creating a photon which decays into an new electron positron pair



Eventually the increasing number of diagrams wins, but still give very good results for the electroweak model.

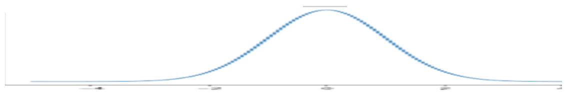
Alternative approaches:

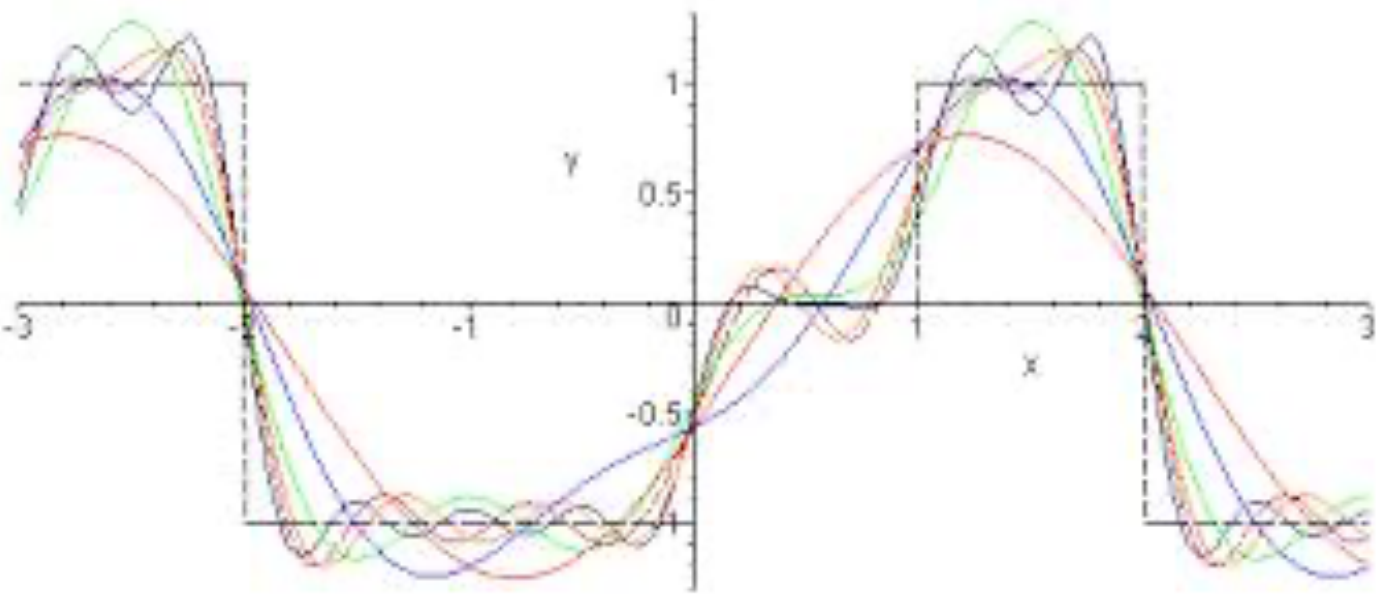
1. Lattice QCD:
    - Spacetime is approximated by a ***discrete*** array of points
    - ***All*** paths on the lattice are combined as an approximation to the path integral.



    - This method has given good predictions (to several percent) of the mass of the proton.

2. AdS/CFT
    - A new approach arising from string theory in which there is a mapping between solutions in QFT and general relativity. (Kevin's colloquium yesterday dealt with a part of this)




y
x
1
0.5
-0.5
-3
-1
0
1
3