1 Variable stars

1.1 Discovery and correlation

First pulsating star: Mira 1595. A long period variable, irregular period between 100 and 700 days.

The first collection of larger numbers of variables was accomplished in Pickering's lab by H. S. Leavitt. She looked explicitly for stars present on one photographic plate and not on another at a different time. After finding 16 variable stars, Leavitt plotted their periods against their absolute magnitudes and found a roughly linear relationship.

Eventually she identified 2400 variable stars; now about 20,000 are known. Improving her original plot by using certain lines in the infrared has led to a very strong correlation between absolute magnitude and period.

What does this let us compute?

Knowing the period of a variable now tells us the absolute magnitude, from which we may compute the distance. Since variable stars are giants, they may be seen from millions of light years away, much much further than we can measure parallax. Thus, Cepheid variables (one of a few kinds) become a standard candle for cosmic distance. It was this realization of Leavitt's, together with observations of Cepheids in the Magellenic Clouds that led to the discovery of *galaxies.*

1.2 Mechanism

After it was realized that the pulse rate for some variables was faster than a hypothesized dark object could orbit them, Shapley hypothesized that the star itself was radially pulsing. This is not an impossible scenario: a sphere of gas has normal modes like a guitar string or an organ pipe. For a star, these will be longitudinal density waves, with the fundamental having a node at the center of the star and the maximum radial displacements at the surface. Higher harmonics have one or more additional nodes between the center and surface.

One problem with the pulsation hypothesis is that the driving pressure cannot arise from the intense heat at the center of the star because the center is a node and there wouldn't be sufficient transfer or energy outward. Eddington hypothesized that at some radius away from the center, an increase in pressure could increase the opacity of the star, thereby trapping heat inside an opaque barrier. As this heat built up, it is argued, it eventually drives the outer layers outward until the excess energy is spent and the star starts to compress again, re-establishing the opaque barrier.

There is a problem with the Eddington hypothesis, but its resolution explains why and where such an opaque layer occurs. Opacity does increase with increasing pressure, but it decreases with increasing temperature. Both pressure and temperature increase together as the star compresses. However, the effect of temperature is greater, so the opacity generally decreases instead of growing.

The resolution occurs by noting the presence of zones of ionization. In a certain temperature range, hydrogen and/or helium become singly ionized. In a zone where this ionization is partial, an increase in pressure is not accompanied by an increase in temperature because the excess energy goes into further ionization. These zones become more opaque, and the Eddington mechanism works.

1.3 Period

A crude estimate of the period gives the right range for oscillations. Suppose (radically!) that we have a star of constant density with pressure given by the mass of the overlying material.

Consider a small cylinder of gas at radius r, of height dr and area A. The difference in gravitational force between the top and bottom must be offset by a difference in pressure:

$$dP = \frac{F_{top} - F_{bottom}}{A} = \frac{GM_r dm}{Ar^2}$$

where M_r is the mass inside radius r and dm is the mass inside the cylinder. Then

$$\begin{aligned} dP &= -\frac{G}{Ar^2} M_r dm \\ &= -\frac{G}{Ar^2} \frac{4\pi\rho r^3}{3} \rho A dr \end{aligned}$$

and the rate of change of pressure with radius is

$$\frac{dP}{dr} = -\frac{4\pi G\rho^2 r}{3}$$

Integrate from r to the surface, where $P = 0$:

$$\begin{aligned} \int\limits_{P(r)}^{0} dP &= -\frac{4\pi G\rho^2}{3} \int\limits_{r}^{R} r dr \\ -P &= -\frac{2\pi}{3} G\rho^2 \left(R^2 - r^2\right) \end{aligned}$$

The velocity of sound in a gas is given by

$$\begin{aligned} v_s &= \sqrt{\frac{\gamma P}{\rho}} \\ &= \sqrt{\frac{2\pi\gamma G\rho}{3} \left(R^2 - r^2\right)} \end{aligned}$$

so the time for a pulse to travel from the center to the surface and back is approximately

$$\begin{aligned} T &= 2 \int\limits_{0}^{R} \frac{dr}{v_s(r)} \\ &= \sqrt{\frac{6}{\pi\gamma G}} \int\limits_{0}^{R} \frac{dr}{\sqrt{R^2 - r^2}} \\ &= \sqrt{\frac{3}{2\pi\gamma G}} \int\limits_{0}^{R} \frac{dr}{R\sqrt{1 - \frac{r^2}{R^2}}} \end{aligned}$$

Let $\frac{r}{R} = \sin\theta$, so that $\frac{dr}{R} = \cos\theta d\theta$. Then

$$\begin{aligned} T &= \sqrt{\frac{6}{\pi\gamma G\rho}} \int\limits_{0}^{R} \frac{dr}{R\sqrt{1 - \frac{r^2}{R^2}}} \\ &= \sqrt{\frac{6}{\pi\gamma G\rho}} \int\limits_{0}^{R} \frac{\cos\theta d\theta}{\cos\theta} \\ &= \sqrt{\frac{6}{\pi\gamma G\rho}} \theta \\ &= \sqrt{\frac{6}{\pi\gamma G\rho}} \left[\sin^{-1} \frac{r}{R}\right]_0^R \end{aligned}$$

$$= \sqrt{\frac{6}{\pi\gamma G\rho}}\frac{\pi}{2}$$

$$= \sqrt{\frac{3\pi}{2\gamma G\rho}}$$

For a star with 5 solar masses and a radius 50 times that of the sun, $\gamma = \frac{5}{3}$ and ρ found from the mass and radius, this works out to a period of about 12 days, which is in the typical range of 1 - 100 days for Cepheids.

2 Supernovae

2.1 Highly variable type O stars

Certain stars are extremely variable and have unusual spectra:

- Luminous blue variables (extreme upper left in HR diagram)
- Wolf-Rayet stars: unusual spectra:
  - WN: He and Nitrogen lines
  - WC: He and Carbon lines
  - WO: Oxygen lines
  - Subclasses within these by degree of ionization

See images.

These are all now seen to be late evolutionary stages of stars of type O stars. In all of these, substantial expulsion of mass occurs when the luminosity pressure exceeds the gravitational attraction. As they expel large amounts of mass from their outer layers, they expose deeper and deeper layers of the star containing different elements.

$M > 85M_\odot$	$O\ supergiant \Rightarrow$	$O : strong\, lines$	LBV	$WR : Nitrogen$	$WR : Carbon$	SN
$40M_\odot < M < 85M_\odot$	$O \Rightarrow$	$strong\, lines$	$WR : N$	$WR : C$	SN	
$25M_\odot < M < 40M_\odot$	$O \Rightarrow$	$Red\ supergiant$	$WR : N$	$WR : C$	SN	
$20M_\odot < M < 25M_\odot$	$O \Rightarrow$	$Red\ supergiant$	$WR : N$	SN		
$10M_\odot < M < 20M_\odot$	$O \Rightarrow$	$Red\ supergiant$	$Blue\ supergiant$	SN		

In each of these classes, SN stands for supernova.

2.2 Supernova observations

Only a three supernovae have been observed within the Milky way galaxy and five anywhere before the use of telescopes:

1. HB9 was marked on star charts from India about 5000 years ago.
2. SN 185 was viewed by Chinese astronomers in 185 CE
3. On about April 30, 1006 a magnitude -9 star suddenly appeared in the constellation Lupus , bright enough to be seen in daytime and to read by at night. It was reported by astrologers in Europe, the Middle East, and Asia.
4. July 4, 1054, Yang Wei-T'e documented a "guest star" in Taurus, which gradually became invisible after a year. It was also observed in Japan, Korea and Arabia. It was also visible in daylight. This supernova is the source of the Crab nebula (Crab supernova remnant). (Milky Way)

5. Tycho Brahe recorded a supernova in 1572. (Tycho's supernova) (Milky Way)
6. Johannes Kepler (Tycho Brahe's student) recorded a supernova in 1604. (Kepler's supernova) (Milky Way)

The closest supernova since Kepler's supernova occured in the Large Magellenic Cloud in 1987. (SN 1987A)

2.3 Types of supernovae

Supernovae are classified by certain distinct spectral characteristics.

- Type I: No Hydrogen lines
  - Type Ia: Strong Si II line at 615 nm
  - Type Ib: Strong Helium lines
  - Type Ic: No strong Helium lines
- Type II: Strong Hydrogen lines