Black Holes

April 11, 2020

Lecture: Black Holes Black Holes

Questions from Christian:

Dr. Wheeler,

I have some questions about your Black Hole slides. 1. What is the difference between the "last lightlike orbit," the Swarchschild radius, and the event horizon? [Slide 5]

2. We know that with linear forces, like electromagnetism, we can use Gauss' law to determine the enclosed charge as related to the electric flux. But with a nonlinear force like gravity, are we only allowed to use such a law in the Newtonian limit?

For a black hole, is the gravitational flux not quite related to the enclosed mass? For example, if the mass distribution was changing, would the gravitational flux also change? I ask because, if a black hole was a singularity, it seems like it would be constantly falling into itself, and maybe that would change the binding energy and, thus, the gravitational flux outside the event horizon?

Answers:

1. What is the difference between the "last lightlike orbit," the Swarchschild radius, and the event horizon? [Slide 5]

This is the difference between orbital velocity and escape velocity. The only way an object can escape from just outside the Schwarzschild radius is to travel at nearly the speed of light straight away from the center.

The last lightlike orbit is where the orbital velocity for a circular orbit equals the speed of light,

For Newtonian orbits, these would be:

- Escape velocity is achieved when a body has enough kinetic energy has to overcome all of the gravitational binding energy, so we need

$$
\begin{aligned}
\frac{1}{2}mv^2 &= \frac{GMm}{r}, \\
v_{escape} &= \sqrt{\frac{GM}{r}},
\end{aligned}
$$

- Orbital velocity for a circular orbit is when the gravitational acceleration just provides the centripetal acceleration needed for a circle,

$$
\begin{aligned}
\frac{v^2}{r} &= \frac{GM}{r^2}, \\
v &= \sqrt{\frac{GM}{r}},
\end{aligned}
$$

These differ by the $\sqrt{2}$. For the general relativity case, it's the same idea but the two differ by a factor of 3/2.

2. We know that with linear forces, like electromagnetism, we can use Gauss' law to determine the enclosed charge as related to the electric flux. But with a nonlinear force like gravity, are we only allowed to use such a law in the Newtonian limit?

Yes, that's exactly right. There's no general result for the amount of energy contained within a given surface; in fact, in curved spaces energy is not easily defined. What we CAN define is energy and momentum when a space is asymptotically flat. That is, far away from gravitating sources, spacetime is flat and there we have the usual symmetries of flat spacetime: translations in space and time, and rotations. With these symmetries, we can define conserved quantities: momentum, energy and angular momentum.

Also, at large distances, gravity is weak enough that we can use the Newtonian limit of general relativity. This is how we establish that the constant in the Schwarzschild solution is $2Gm/c^2$.

3. For a black hole, is the gravitational flux not quite related to the enclosed mass? For example, if the mass distribution was changing, would the gravitational flux also change? I ask because, if a black hole was a singularity, it seems like it would be constantly falling into itself, and maybe that would change the binding energy and, thus, the gravitational flux outside the event horizon?

The idea of a black hole "constantly falling into itself" is the wrong picture. An isolated black hole - one without an accretion disc - very quickly reaches equilibrium. The field outside is static and given by Schwarzschild, with the mass constant. For a rotating black hole, the solution is modified by rotational terms, but also is unchanging. The only way to change the exterior field of a black hole is to throw something in.

If we have a static black hole and drop something radially in, then after a (very short!) finite time it is again static with a slightly larger mass. During the brief transition, there will be some gravitational waves produced.

Think of a black hole the way you think of any isolated mass. It has a gravitational field, and things fall in.

Questions from Brock

Brock On the black hole presentation, slide 24 you talk about the laws of thermodynamics of black holes. You mention surface gravity. Is this surface gravity the gravity felt at the surface of the black hole analogous to how we feel gravity on the surface of the earth? Also if that is the case, what is meant by the 3rd law saying that it cannot go to zero? Why would that need to be stated as a law when it seems pretty basic that anything with mass has intrinsic gravity?

Yes, that's right. Surface gravity is another term for gravitational acceleration, like $g = 9.8$ on Earth. For a black hole, the surface gravity is the gravitational acceleration at the event horizon. Surprisingly, for a black hole it is *inversely* proportional to the mass:

$$\kappa = \frac{c^2}{4GM}$$

The acceleration decreases with mass because the horizon of a larger mass black hole lies further out.

The formula is more complicated for a rotating or charged black hole, such that there are limiting values for the angular momentum and charge.

For the 3rd law, remember the usual 3rd law of thermodynamics. Basically it says that you can never reach absolute zero; there is always some finite entropy to a system. If you could get to absolute zero temperature, you could build a perfectly efficient engine, which experience tells us we can't do.

For a black hole, zero surface gravity corresponds to infinite mass in the Schwarzschild case, but for rotating or charged black holes it can occur for finite values of mass, charge, and angular momentum. I suspect, without looking into it in detail, that the condition again insures that we can't build a perfectly efficient engine.

Brock On your cosmology presentation, on slide 20 you bring up how inflation solves the problem of why we dont see magnetic monopoles. But does this inflation shed any light on how they are formed or in their validity?

I enjoy the quotation from Martin Rees: "Skeptics about exotic physics might not be hugely impressed by a theoretical argument to explain the absence of particles that are themselves only hypothetical. Preventive medicine can readily seem 100 percent effective against a disease that doesn't exist."

It's true – anything about monopoles is conjectural. That said, there are plausible reasons to worry about magnetic monopoles. They're not predicted or forbidden by inflation, but they ARE predicted by most unified field theories. If there is some unification of the fundamental interactions (strong with electroweak, in particular) then despite the enormous mass such theories predict for magnetic monopoles, the conditions of the early Big Bang would produce them. Once produced, they must be stable by conservation of (magnetic) charge. If that much is true, then inflation gives a way to understand why we don't see them - they would have been produced before inflation, so inflation would spread them to a very low density in the later universe.